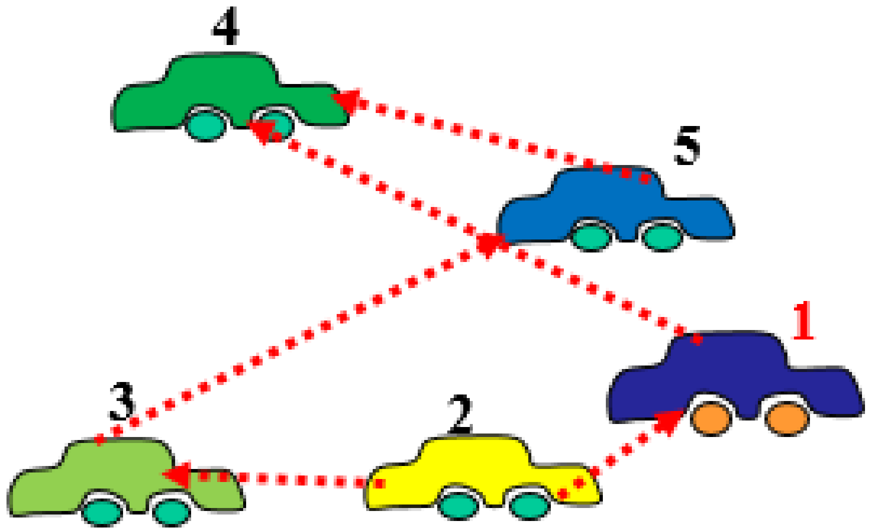
Code Division Multiple Access

- **Multiple Access** is the use of multiplexing techniques to provide communication service to **multiple users** over a **single channel**.
- **Channel Allocation**: The coordination of the usage of a single channel among multiple **source – destination** pairs.
- The algorithm which implements the channel allocation are called **medium access control** (MAC) or multiple access protocols.
- **Medium Access Control protocols** can be classified into
  - Conflict free protocols:
  - Random access protocols:

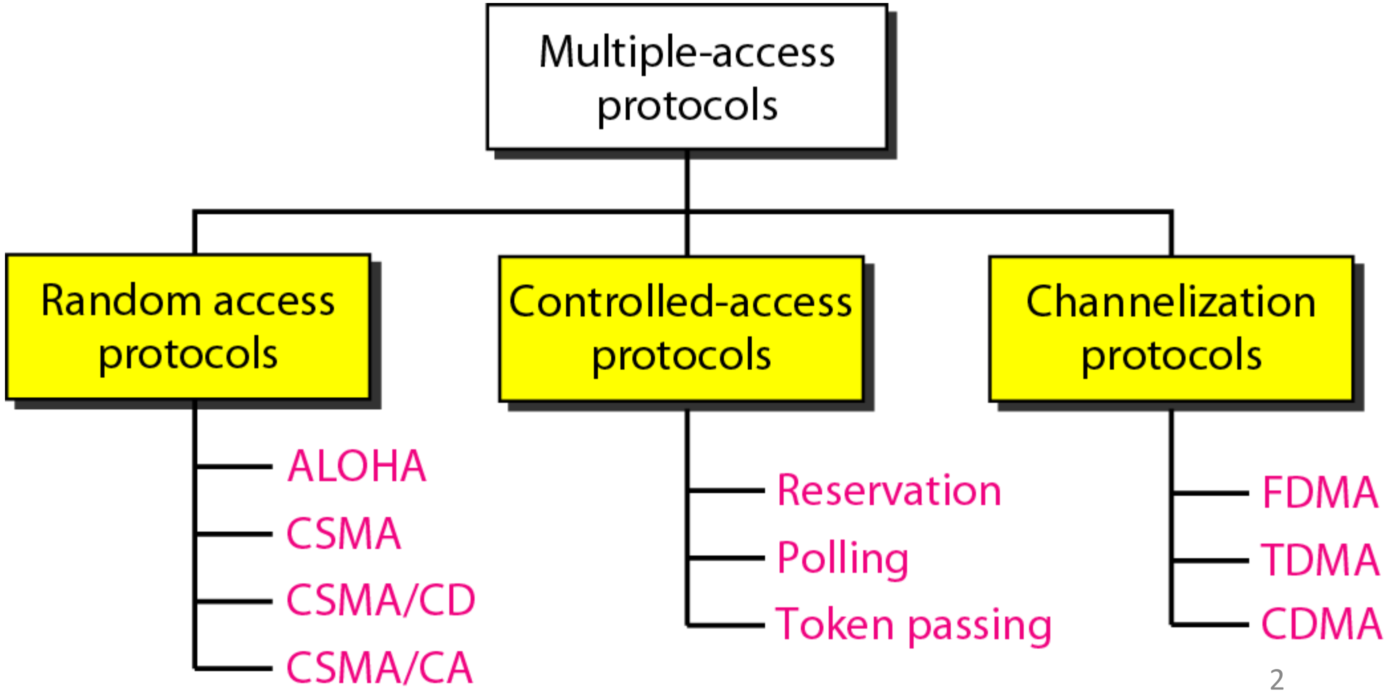


Multiple Access Protocols

- A **Conflict free protocols**: Collisions are completely avoided by allocating the channel access to sources in a predetermined manner. Examples are TDMA, FDMA and CDMA. This is equivalent to circuit switching and is inefficient for bursty type of loads.

- **Random access protocols**: These are classified as contention systems where the stations compete to access the channel. The contention could be completely random or controlled.
- Collisions can occur between transmitted packets of different users trying to access the channel.
- A collided packet has to be transmitted until it is received properly at the destination.

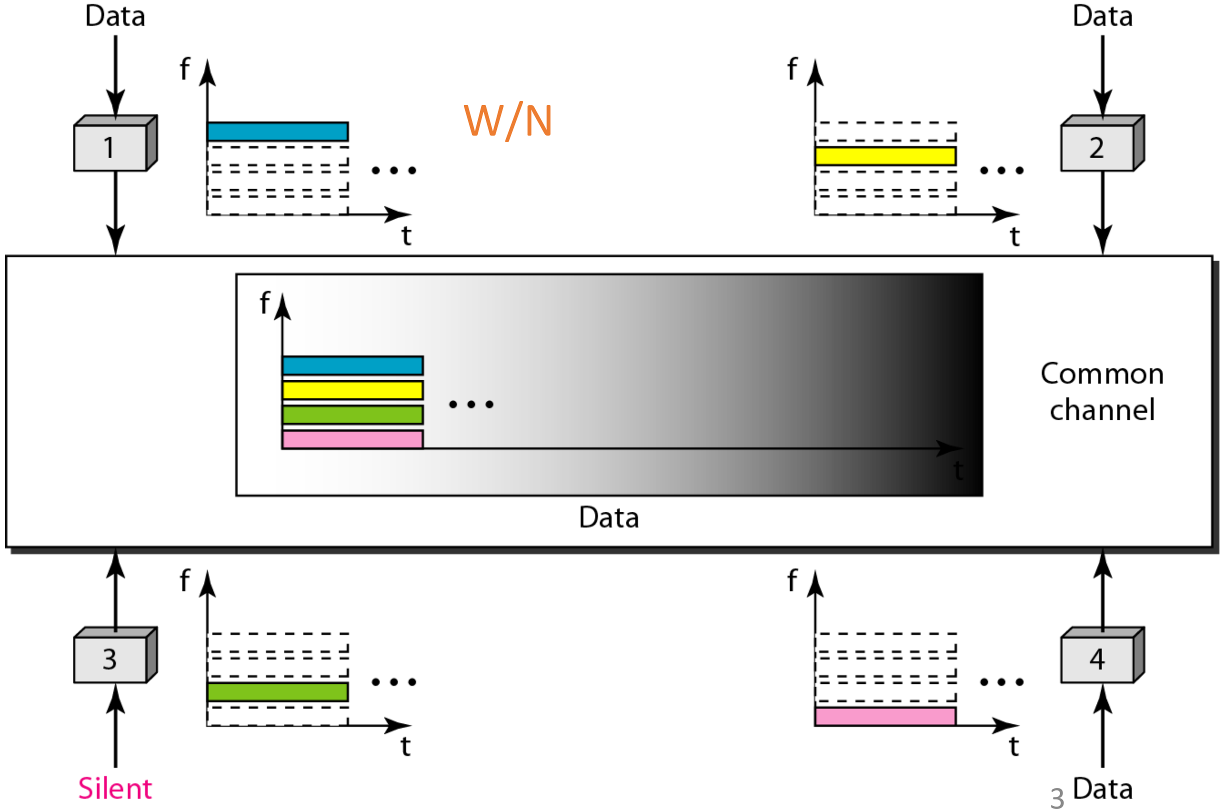
In this lecture, we will address conflict-free protocols only.



Frequency Division Multiple Access (FDMA)

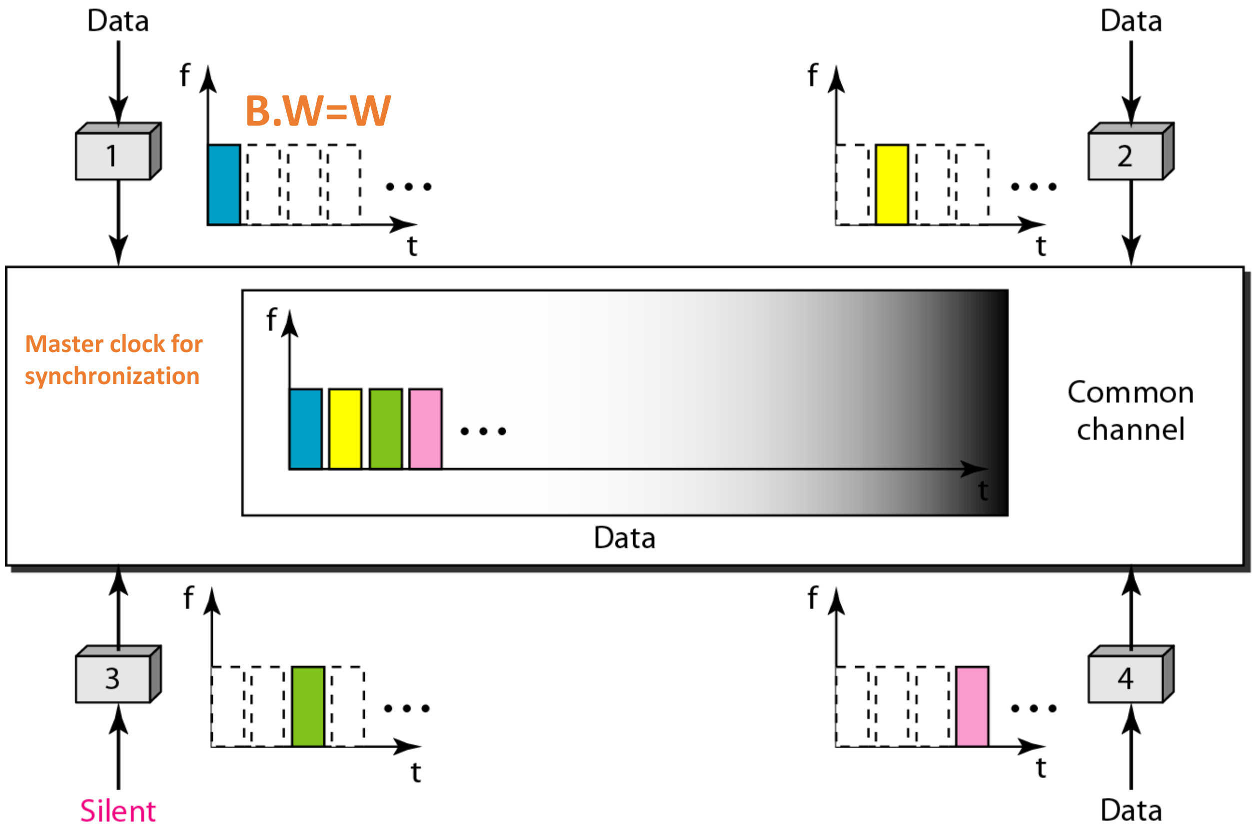
- Let W be the channel bandwidth and let N be the number of users. Each user is allocated W/N of the available B.W for his exclusive use.
- Each user can transmit at any given time provided he uses his own band. Collisions are completely avoided.
- FDMA was used in the first generation of mobile systems known as Advanced Mobile Phone System (AMPS).

- **Advantages**:
  - FDMA is simple and **efficient**, especially, when the number of sources is small (and constant) and each user has data to send.
  - It does not require coordination between stations.
- **Disadvantages**:
  - FDMA is **inefficient** when the number of sources is large and varying.
  - It is also inefficient when the sources send the information in a bursty manner; channel is underutilized.
  - Some stations may not have data to send while others have bursty data.



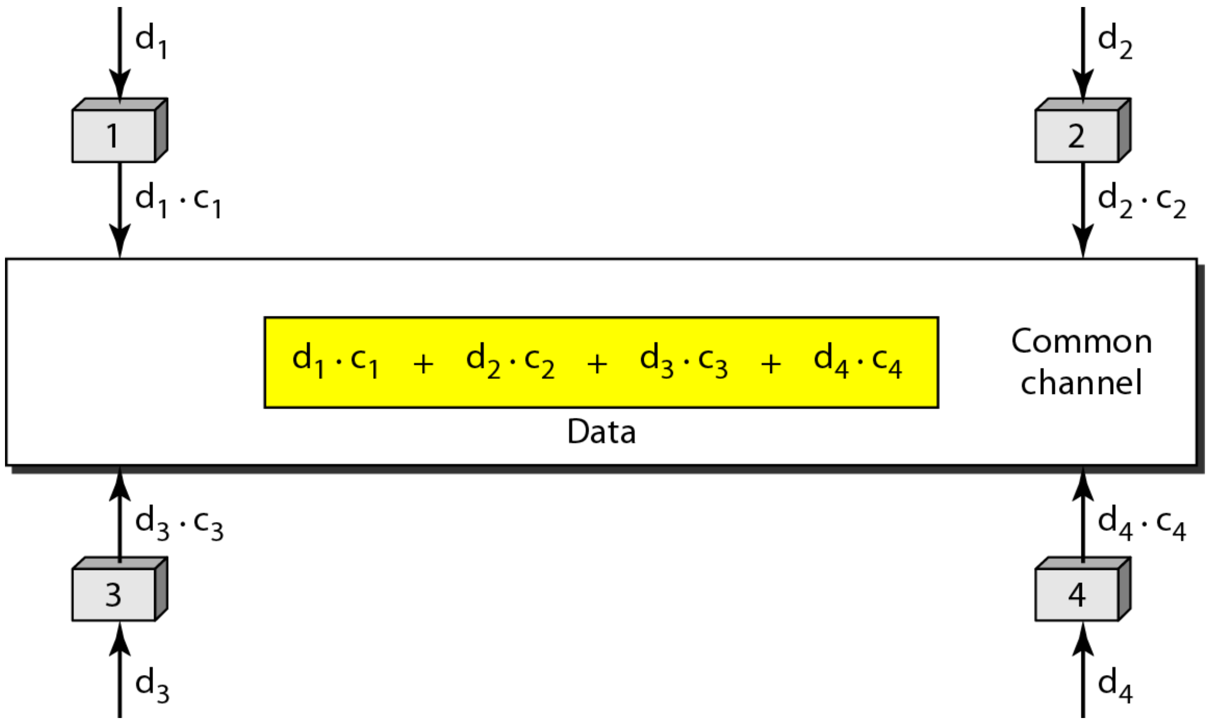
Time Division Multiple Access (TDMA)

- Let N be the number of sources. The time axis is divided into N slots and each slot is allocated to a source.
- Each source transmits only during its slot, avoiding the possibility of a collision.
- When a user transmits during its slot, it utilizes the entire B.W. of the channel and this B.W. will be made available to the next user during the succeeding time slot.
- The collection of the N slots is called a **cycle**.
- TDMA requires some form of synchronization.
- TDMA suffers from the same disadvantages as FDMA, namely, the underutilization when the sources have intermittent and inactive data sources.



Code Division Multiple Access (CDMA)

- Let N be the number of sources.
- The N users **occupy the same frequency band** and **transmit / receive messages simultaneously in time**.
- Different users are distinguished by distinct codes assigned to them.
- CDMA relies on a technique called **spread spectrum** in which the transmitted signal occupies a bandwidth much larger than the BW of the message
- The third generation (3 G) of mobile communications uses CDMA.
- The data of each user is spread by a unique **code or chip** , called the **signature waveform.**
- The signature waveforms have to be **orthogonal.**



$$\int_0^{T_b} c_i^2(t)dt = T_b$$

$$\int_0^{T_b} c_i(t)c_j(t)dt = 0; \quad i \neq j$$

Concept of Spread Spectrum

- Let W_1 be the bandwidth of signal $x_1(t)$ **narrowband signal**
- W_2 be the bandwidth of signal $x_2(t)$; **wideband signal**
- W be the bandwidth of signal **$y(t) = (x_1(t))(x_2(t))$;**
- Where **y is a product of two time functions**
- Since this is a multiplication in the time domain, then the spectrum of y(t) is a convolution in the frequency domain.

  $$Y(f) = X_1(f) * X_2(f)$$

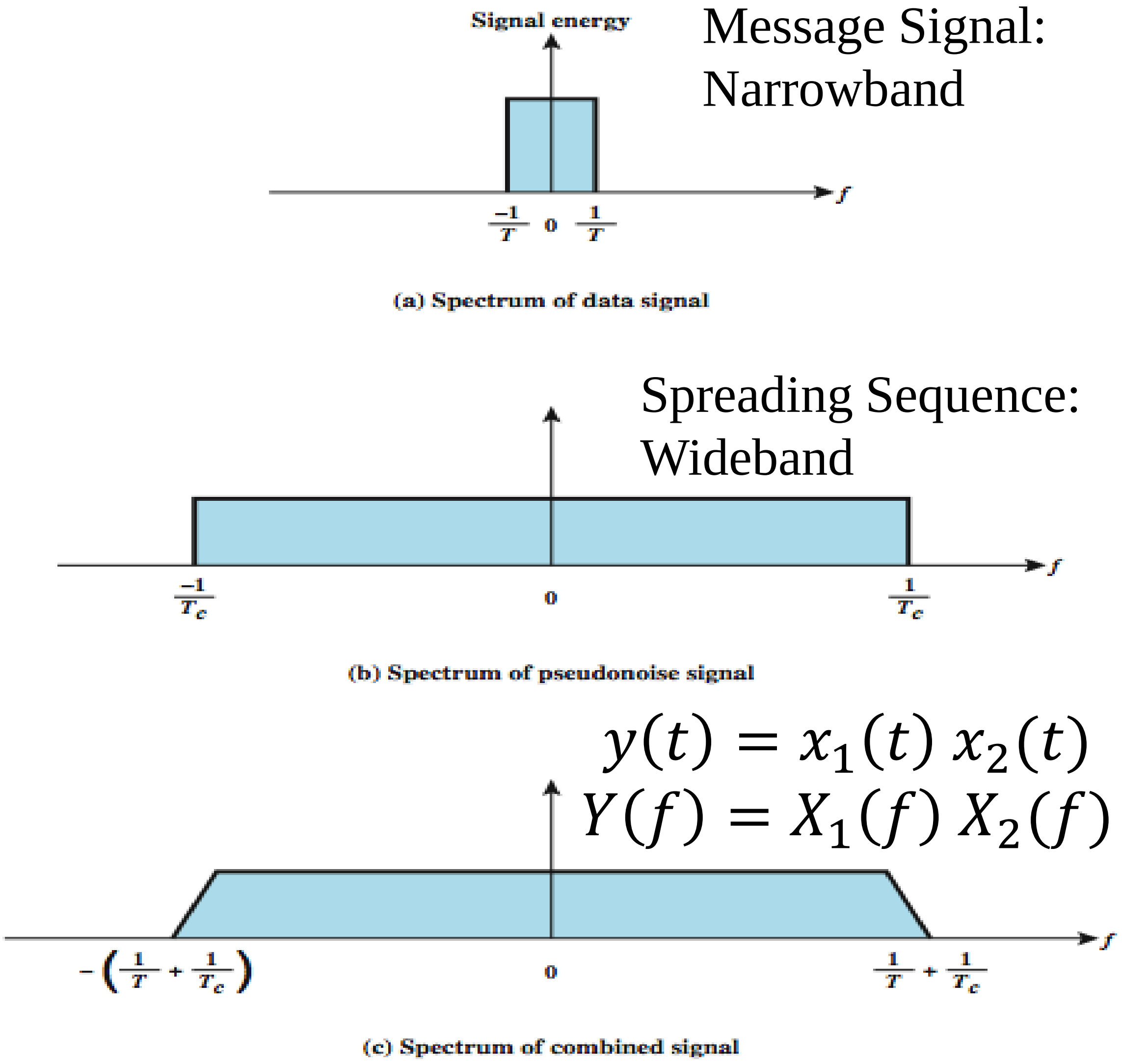
- The bandwidth of y(t)

  $$W = W_1 + W_2 \text{ (B.W=sum of B.W's)}$$

- If $W_2 >> W_1$, then

  $$W \approx W_2$$

- **Conclusion**: When a narrowband signal is multiplied by a wideband signal, the result is a wideband signal.



Spread Spectrum Modulated Signal: Wideband

Spread Spectrum and CDMA

Spread spectrum:

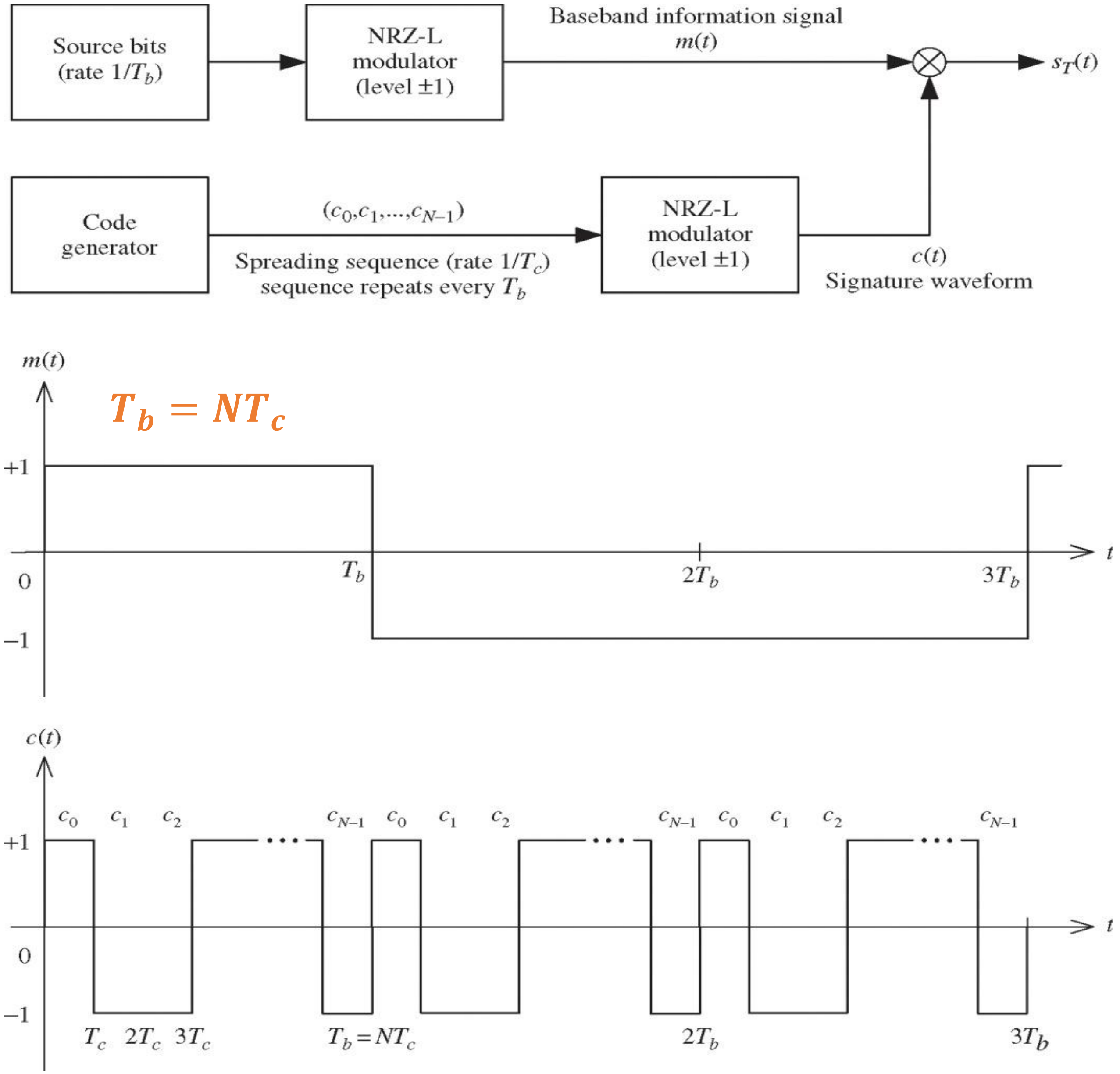
- A technology in which the bandwidth of a signal is spread before transmission.
- Distinct advantages of being **secure** and **robust** against intentional interference (jamming). Will investigate this in the next video lecture.
- Applicable to digital as well as analog signals because both can be modulated and "spread".
- It is the digital applications, in particular, CDMA that made the technology popular in various wireless data networks.

Two ways of implementing spread spectrum: **frequency hopping** (to be presented later) and **direct sequence** (to be discussed in this video lecture)

- **Frequency-hopping spread spectrum:**
  - is a digital communication technique in which the carrier frequency of a signal is **varied among many distinct frequencies in a (pseudo-) random fashion within a wideband channel.**
  - The wideband channel is subdivided into small sub-bands. Multiple access is supported as carrier frequencies of multiple users hop over these narrow bands in a predetermined manner Examples: Bluetooth, an option of IEEE 802.11 wireless LANs.
- **Direct-sequence spread spectrum:**
  - The signal is generated by multiplying the data with a (pseudo-) random sequence so that the resultant rate (chip rate) is high, resulting in a wideband signal.
  - Multiple access is supported as random sequences used by multiple users have low correlations so that the interference due to other-user signals is reduced (but not eliminated).
  - Examples: IS-95, WCDMA, cdma2000

Spread Spectrum: Basic Transmitter and Receiver: Single User

- This slide shows the block diagram of a spread spectrum transmitter (multiplying by the carrier signal has been removed for simplicity in the presentation)
- The source produces data at a rate of $r_b=1/T_b$ bps.
- This is converted to a polar NRZ baseband signal m(t).
- Each bit in m(t) is multiplied by a code **$C=(c_0$ $c_1 \ldots c_{N-1})$.** This is also converted into a polar NRZ waveform (t)
- We maintain that **$T_b = N\ T_c$** . N is an integer.
- T_c : is the duration of each pulse in the code.
- Effect of modulation is to increase bandwidth of signal to be transmitted.



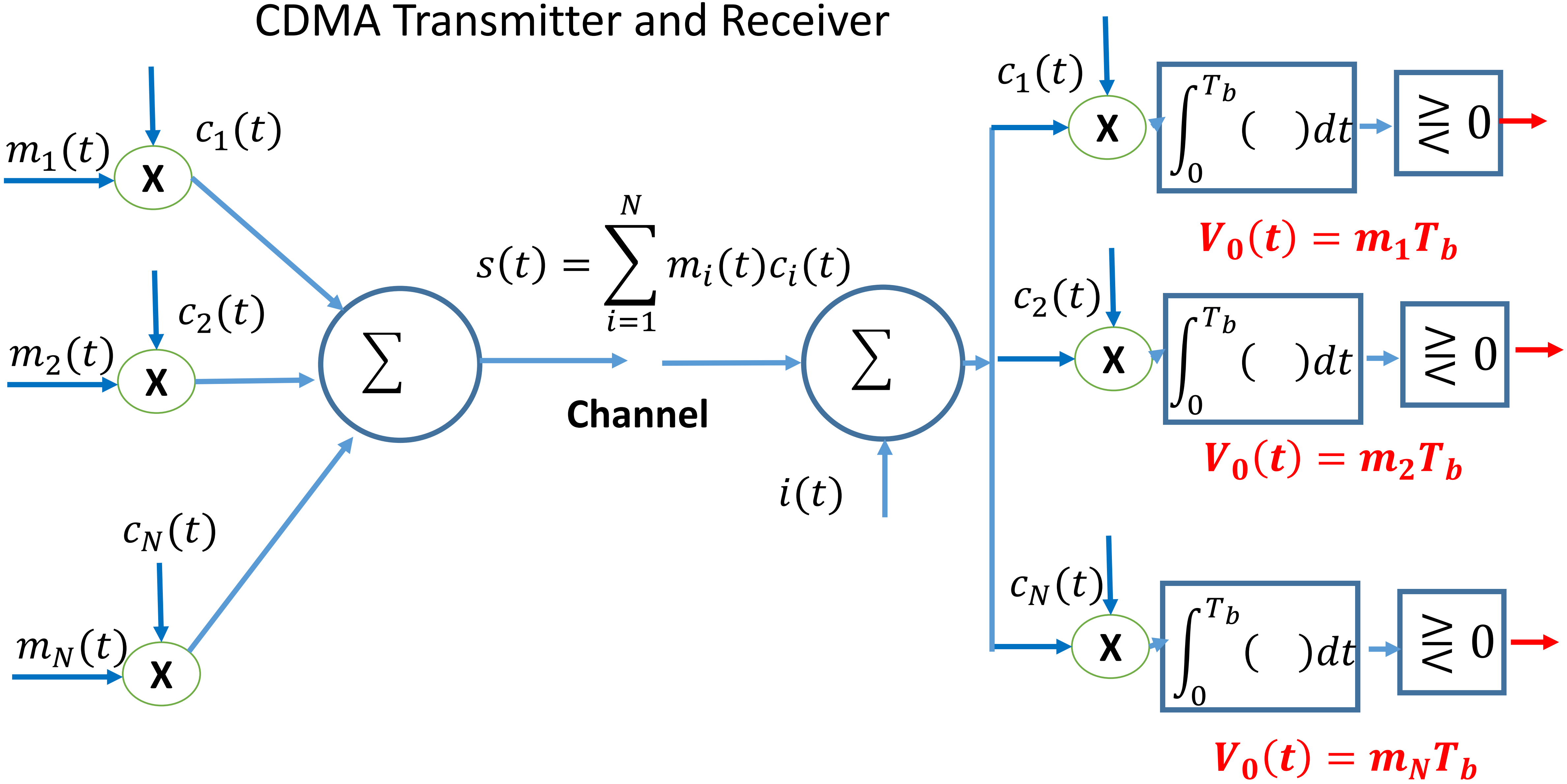
Code Division Multiple Access (CDMA)

- Consider a system where N users have data to send over a common channel
- All users **share the same frequency band** and **transmit simultaneously in time**
- The data of each user is spread by a unique code or chip , called the **signature waveform.**
- The signature waveforms have to be **orthogonal.**

$$\int_0^{T_b} c_i(t)c_j(t)dt = 0; \quad i \neq j \qquad \int_0^{T_b} c_i(t)c_i(t)dt = T_b; \quad i = j$$

- Used mostly in wireless broadcast channels (cellular, satellite, etc)
- ***Encoded signal*** = (message signal) X (signature waveform)
- ***Decoding:*** inner-product of encoded signal and signature waveform
- Allows multiple users to "coexist" and transmit simultaneously with minimal interference (due to the orthogonality of signature waveforms)

CDMA Transmitter and Receiver



Transmitter and Receiver: Noise-Free System

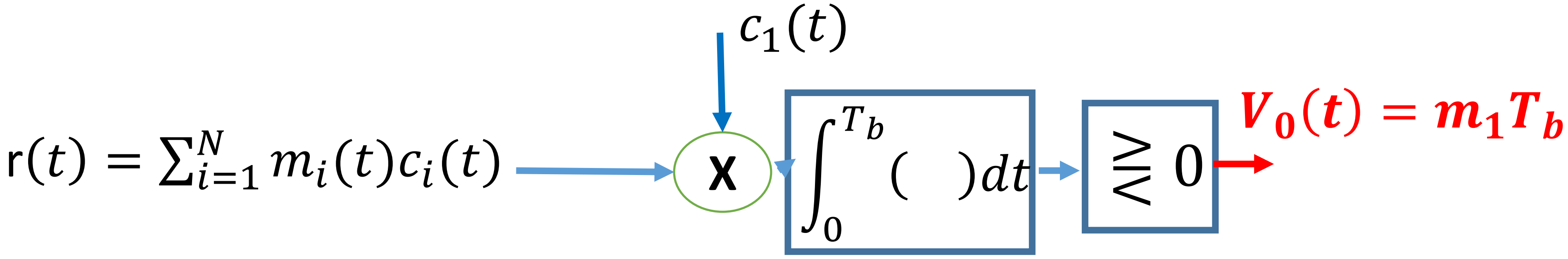
- Transmitted signal s(t) over one bit interval T_b is: $s(t) = m_1 c_1(t) + m_2 c_2(t) + \ldots + m_N c_N(t)$
- The received signal $\mathrm{r}(t) = m_1 c_1(t) + m_2 c_2(t) + \ldots + m_N c_N(t)$ (**same as transmitted signal**)
- To demodulate m_1 , for example, we multiply both sides by c_1 and integrate over T_b .

$$\int_0^{T_b} r(t) c_1(t) dt = \int_0^{T_b} c_1(t)[m_1 c_1(t) + m_2 c_2(t) + \ldots + m_N c_N(t)] dt$$

$$V_0(T_b) = m_1 \int_0^{T_b} c_1^{\,2}(t) dt + \int_0^{T_b} c_1(t)[m_2 c_2(t) + \ldots + m_N c_N(t)] dt = m_1 T_b$$

Output = **desired signal term** (Perfect Orthogonality between signature waveforms)

$$V_0(T_b) = m_1 T_b \qquad m_1 = V_0(T_b) / T_b$$



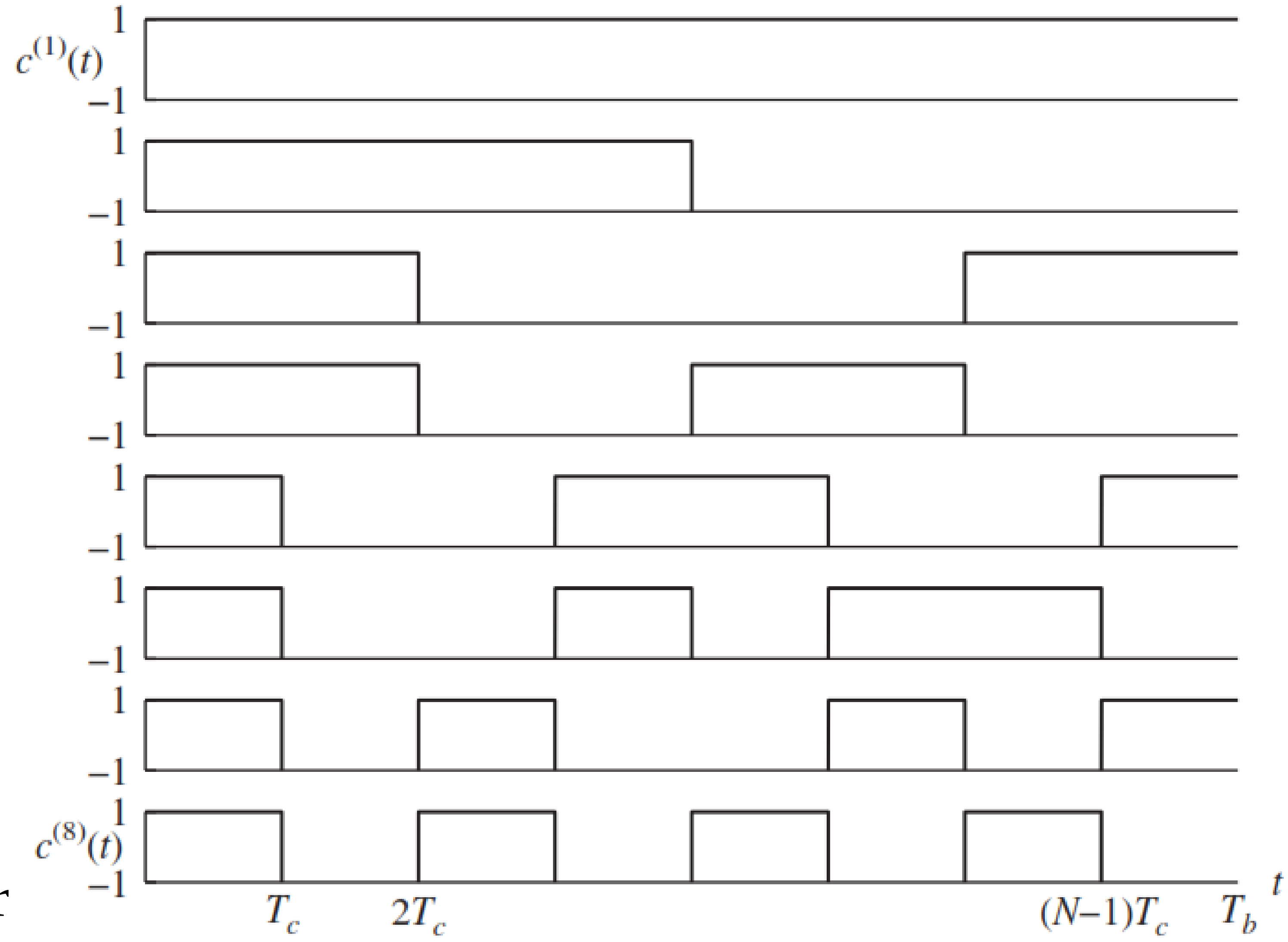
Walsh-Hadamard Codes

- Note that perfect coherence is needed at the receiver side to eliminate the multi-access interference (MAI) term (to be explained later)
- One possible set of orthogonal waveforms (used **at the base station of a mobile system in the downlink**) can be generated using what is known as the Walsh-Hadamard sequences.
- These are codes of length *n* consisting of the n rows of the Walsh-Hadamard matrix:
- Example: For H_2 the codes are: (1, 1), (1, -1)
- For H_4 the codes are: (1, 1, 1, 1), (1, -1, 1, -1), (-1, -1, 1, 1), (-1, 1, 1, -1)
- **Orthogonality**: Every row is orthogonal to every other row.
- Requires tight synchronization
- **Problem**: Cross correlation between different shifts of Walsh sequences is (poor) not zero.

$$H_2 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad H_4 = \begin{pmatrix} H_2 & H_2 \\ H_2 & \overline{H_2} \end{pmatrix} \quad H_8 = \begin{pmatrix} H_4 & H_4 \\ H_4 & \overline{H_4} \end{pmatrix} \quad H_{2n} = \begin{pmatrix} H_n & H_n \\ H_n & \overline{H_n} \end{pmatrix}$$

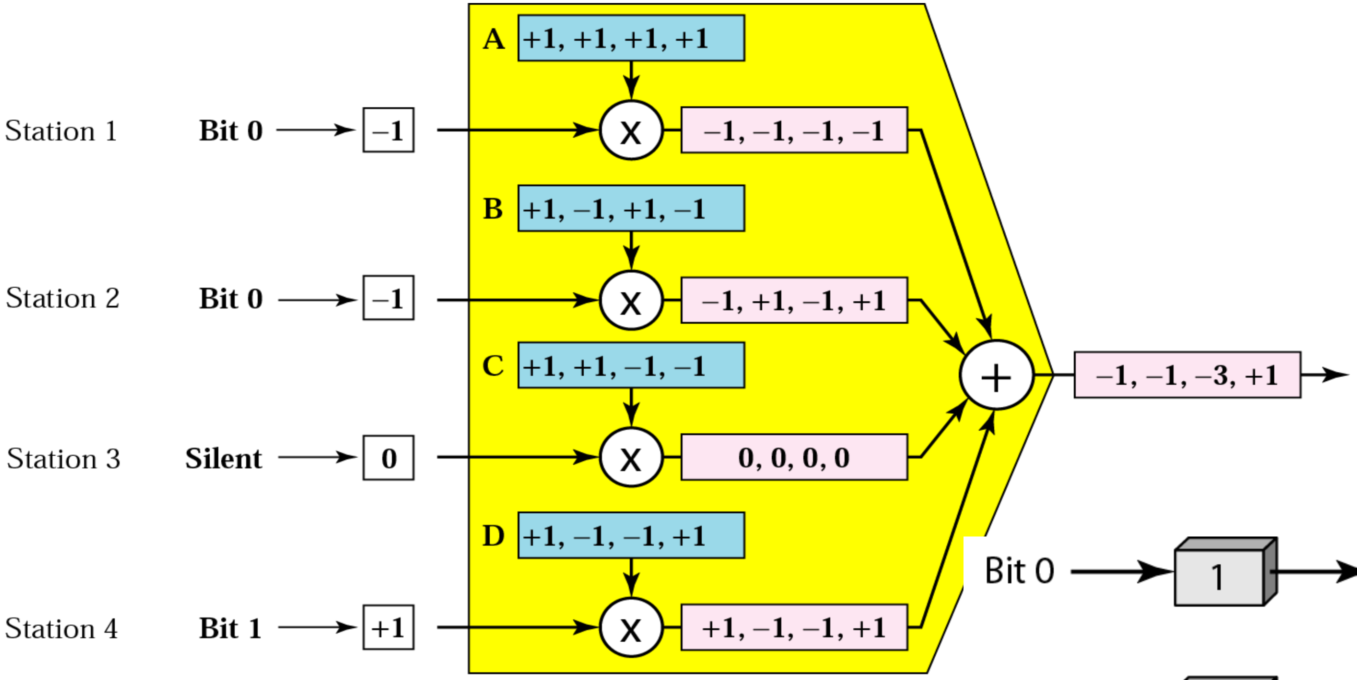
Walsh-Hadamard Code of Length 8

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}$$

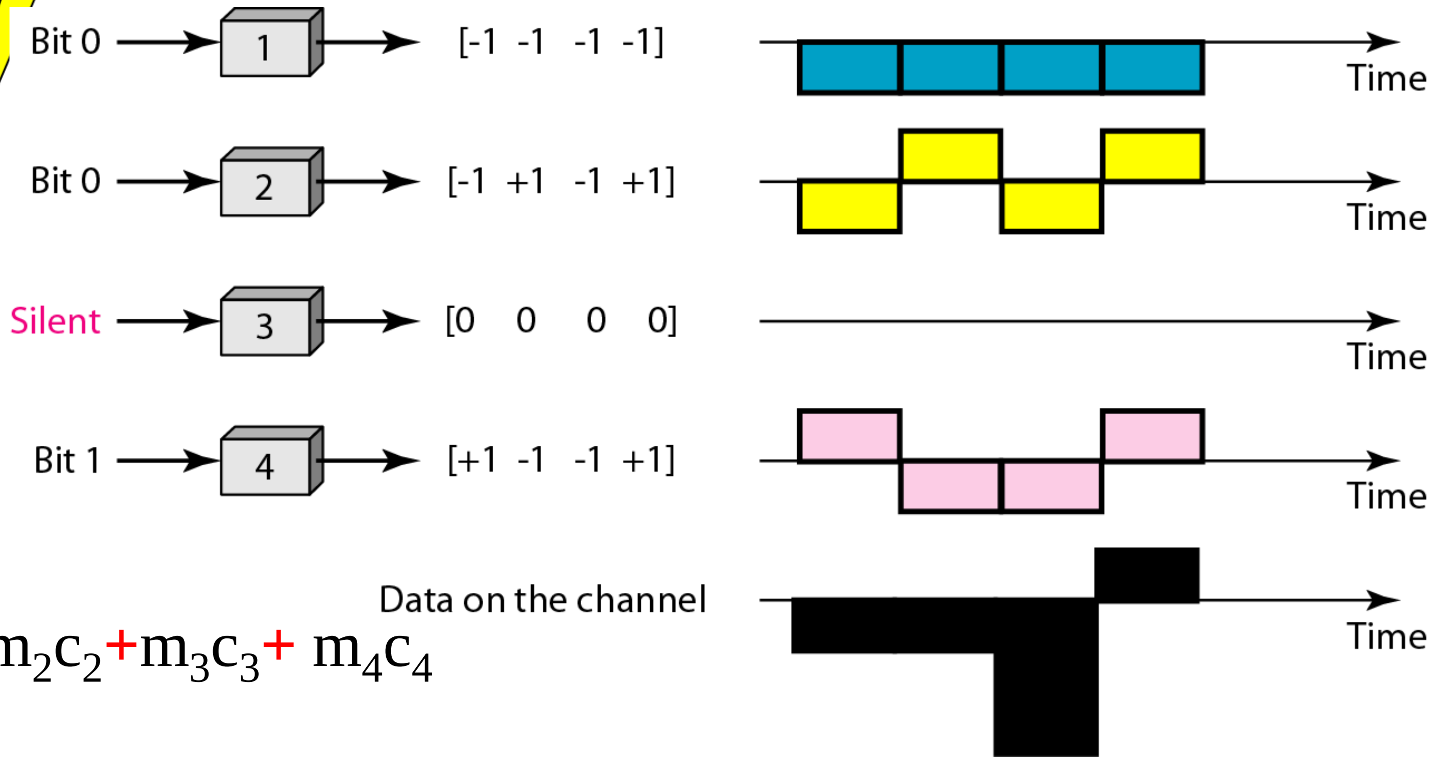


Note that in a Walsh-Hadamard *n x n matrix*, *any row* differs from any other row in exactly *n/2 positions*

Example: CDMA Encoding Details



$$s(t) = m_1c_1 + m_2c_2 + m_3c_3 + m_4c_4$$



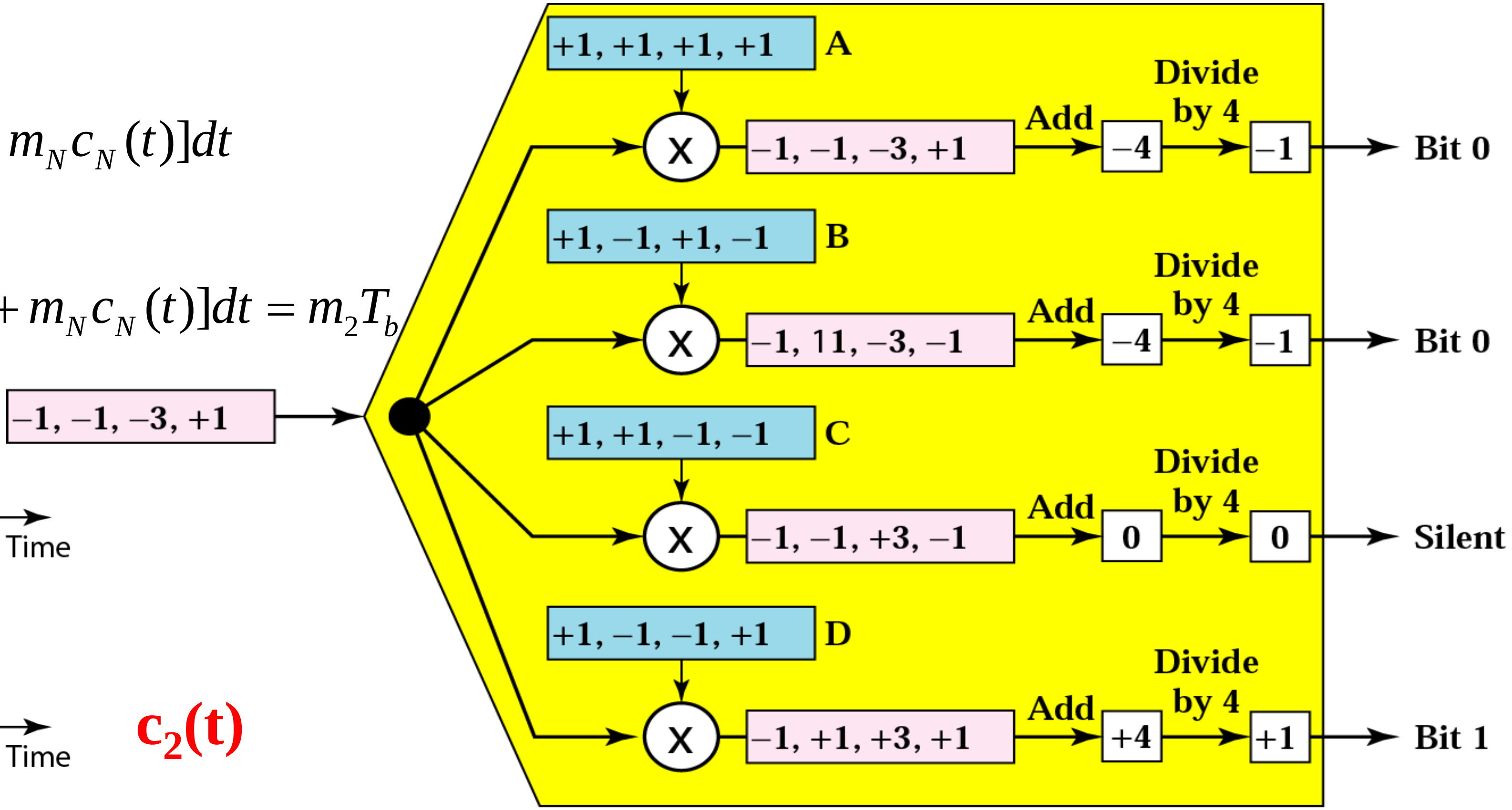
$$s(t) = m_1c_1 + m_2c_2 + m_3c_3 + m_4c_4$$

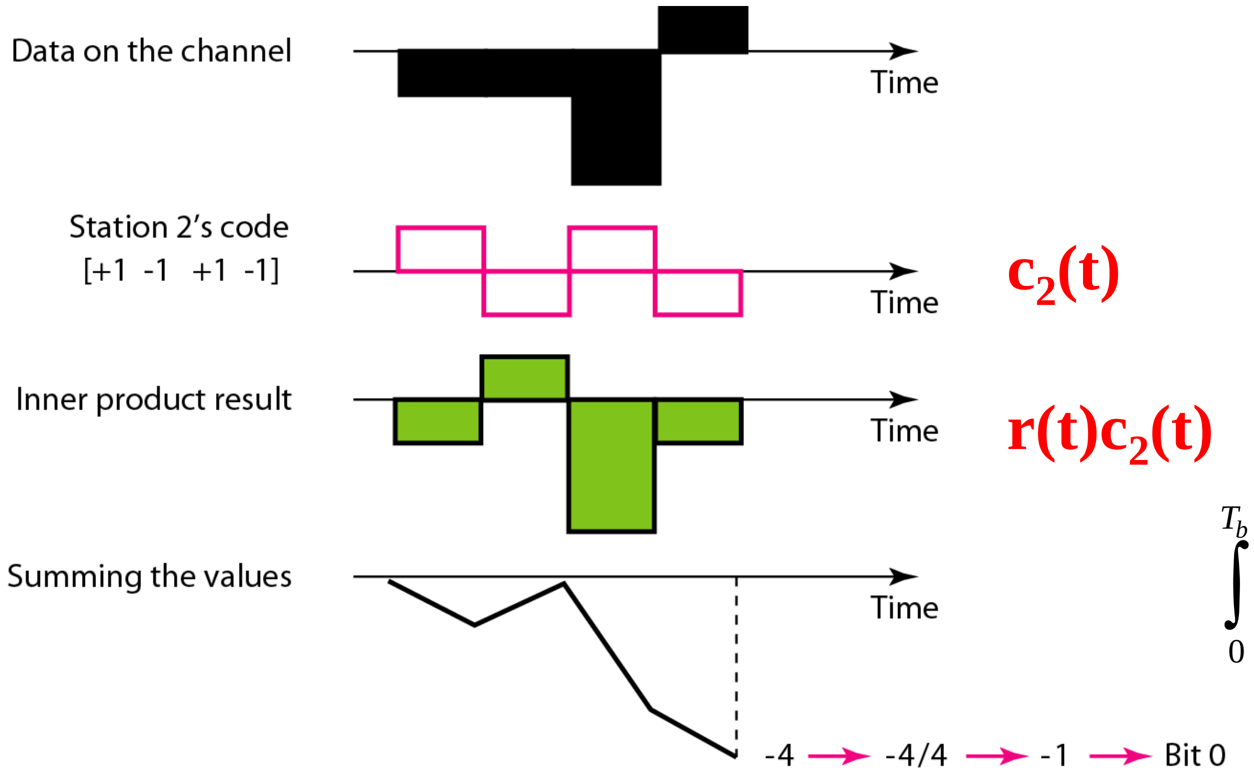
Example: CDMA Demodulation

$r(t)=s(t)=m_1c_1+m_2c_2+m_3c_3+m_4c_4$

$$\int_0^{T_b} r(t)c_2(t)dt = \int_0^{T_b} c_2(t)[m_1c_1(t)+m_2c_2(t)+\ldots+m_Nc_N(t)]dt$$

$$V_0(T_b) = m_2\int_0^{T_b} c_2^{\,2}(t)dt + \int_0^{T_b} c_2(t)[m_2c_2(t)+\ldots+m_Nc_N(t)]dt = m_2T_b$$





$c_2(t)$

$r(t)c_2(t)$

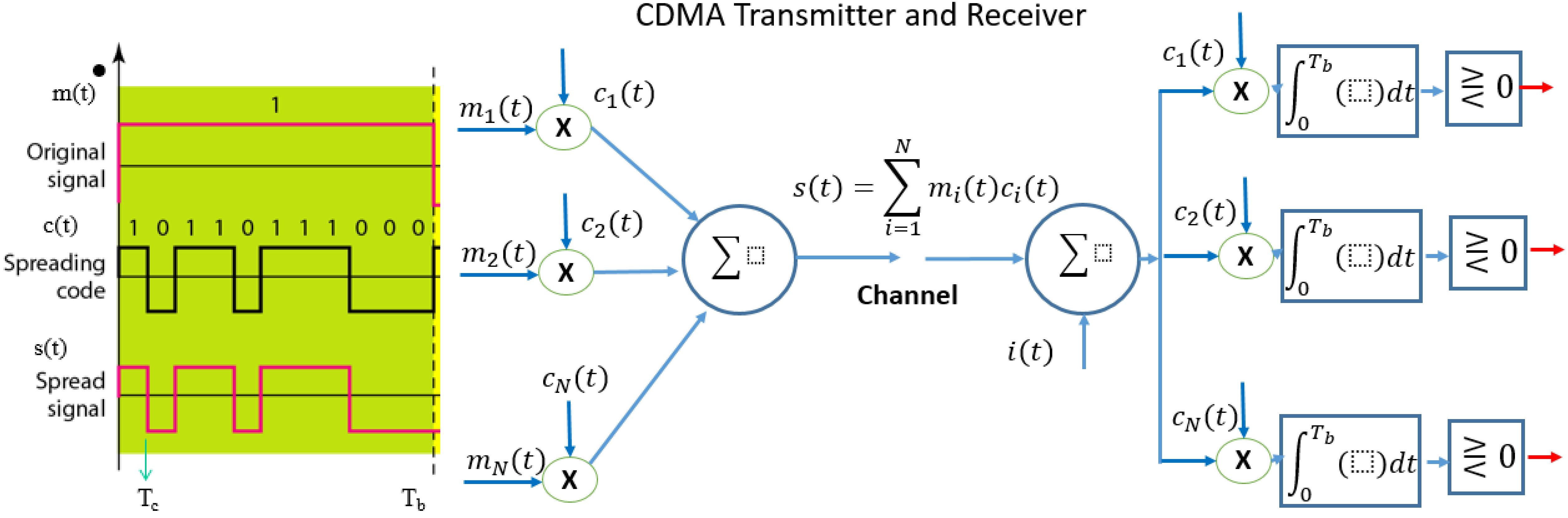
$$\int_0^{T_b} r(t)c_2(t)dt$$

$$m_2 = V_0(T_b)/T_b$$

Jamming and Inter-symbol interference in CDMA

- In this lecture and the next one, we will address two main issues related to CDMA:
  - **First**: Immunity from Jamming Noise and the Reduction of Multi-access interference.
  - **Second**: Desired properties of autocorrelation and cross correlation functions of the spreading sequences.
- Below, are the transmitting and receiving parts of a spreading sequence CDMA system

CDMA Transmitter and Receiver



$T_b = N\ T_c$

Problem 1: Spread Spectrum and Interference Reduction

Here we assume the presence of a jamming signal **i(t)** that occupies the same bandwidth as the message signal, say $m_1(t)$. The received signal is given as:

$$\int_0^{T_b} c^2(t)dt = T_b$$

$$\int_0^{T_b} c_i(t)c_j(t)dt = 0; \quad i \neq j.$$

$$s(t) = m_1 c_1(t) + m_2 c_2(t) + \ldots + m_N c_N(t) + \mathrm{i(t)}$$

$$V_0(T_b) = \int_0^{T_b} [m_1 c_1(t) + m_2 c_2(t) + \ldots + m_N c_N(t) + \mathrm{i(t)}] c_1(t)dt$$

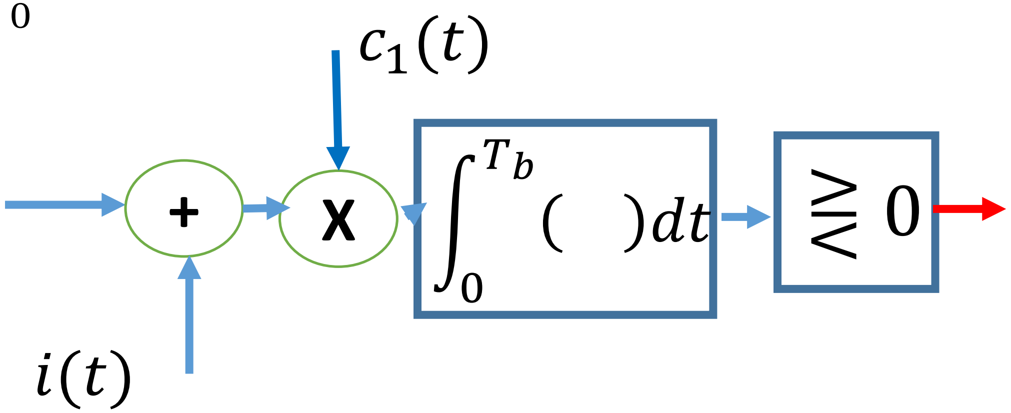
= 1 = 0

$$= m_1 \int_0^{T_b} c_1^{\,2}(t)dt + \int_0^{T_b} [m_2 c_2(t) + \ldots + m_N c_N(t)] c_1(t)dt + \int_0^{T_b} i(t)c_1(t)dt$$

$$= m_1 T_b + \int_0^{T_b} i(t)c_1(t)dt$$

Here, we assume perfect orthogonality among the signature functions.

$$s(t) = \sum_{i=1}^{N} m_i(t)c_i(t)$$

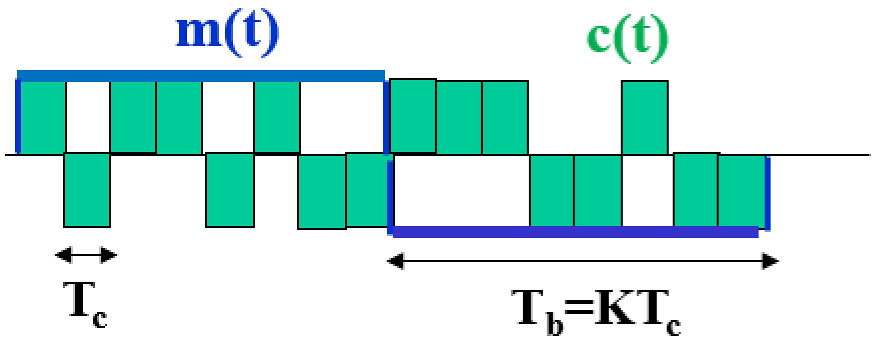


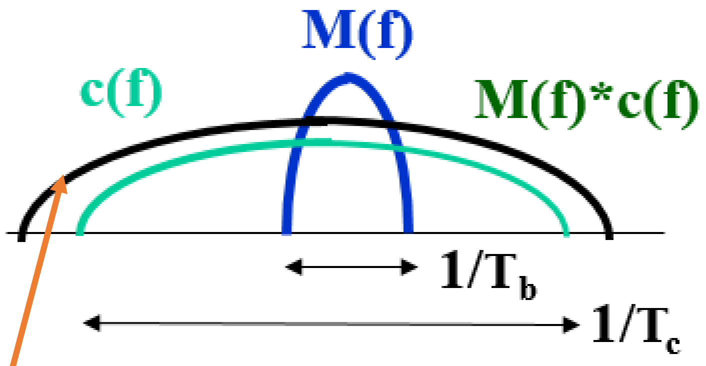
$$V_0(T_b) = m_1 T_b + \int_0^{T_b} i(t)c_1(t)dt$$

Spreading and De-spreading of Message and Noise

Transmitting Side:

Time-domain



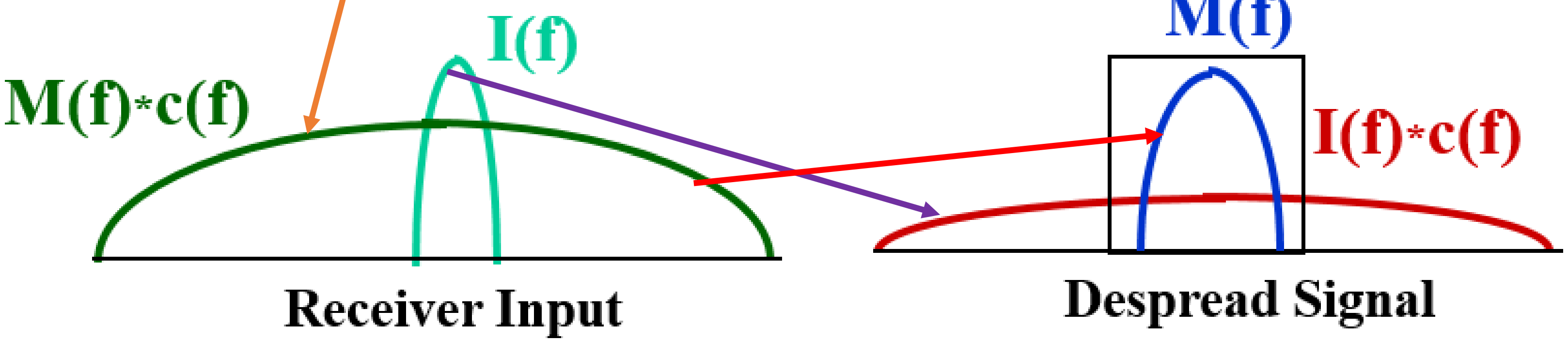


Frequency -domain

When the bit sequence modulates the chip sequence, the spectrum is spread by a factor (G) = T_b / T_c = N

Receiving Side:

$$V_0(T_b) = m_1 T_b + \int_0^{T_b} i(t) c_1(t) dt$$



- The effect of multiplying the interference signal at the receiver side with the spreading sequence is to spread the spectrum of the interference noise over the wide bandwidth.
- The signal component returns to occupy the message bandwidth.
- Only a fraction of the noise power that falls within the message bandwidth is admitted at the receiver output

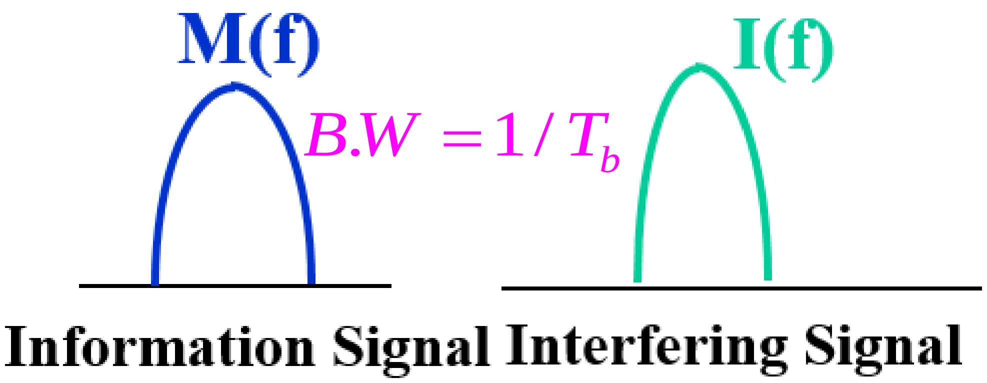
Probability of Error Improvement using the Spreading Sequence

Signal to Noise Ratio without Spreading

Let P_M be the power in the message m(t) and

Let P_I be the power in the in the interfering signal i(t). So,

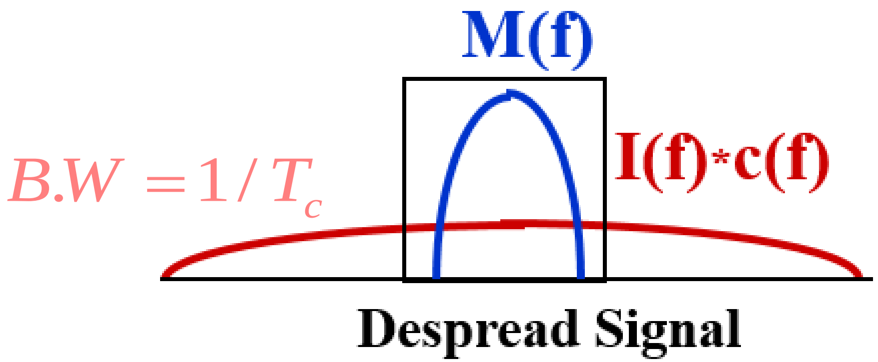
$$SNR_{without} = P_M/P_I$$



Signal to Noise Ratio without Spreading

The effect of multiplying the interference signal at the receiver side with the spreading sequence is to spread the spectrum of the interference noise over the wide bandwidth. The signal component returns to occupy the message bandwidth. Only a fraction of the noise power that falls within the message bandwidth is admitted at the receiver output. Therefore,

$$SNR_{with} = \frac{P_M}{(P_I|N)} = N(P_M/P_I)$$



$$V_0(T_b) = m_1 T_b + \int_0^{T_b} i(t) c_1(t) dt$$

SNR is increased by the spreading factor $G = N = T_b/T_c$. Therefore, spreading reduces the effect of any interfering signal by G and hence reduces the bit error probability.

Two More Problems in Wireless CDMA Systems

Two more problems are encountered in wireless transmission

- The first is Intersymol Interference (ISI)
- The second is Multi-access interference (MAI)
- In the next slides we will address each one of these two problems.
- Moreover, we will explore the desirable properties of the signature waveforms that must be employed to minimize (or eliminate) those kinds of interferences.
- Specifically, we will find the relationship of the ISI with the autocorrelation function and the relationship of the MAI with the cross-correlation functions.
- Let us start by defining the two correlation functions

Two Problems in Wireless CDMA Systems

- Let us start by defining the two correlation functions

Autocorrelation Function

$$R_{ii}(\tau) = \frac{1}{T_b}\int_0^{T_b} c_i(t)c_i(t-\tau)dt$$

Cross Correlation Function

$$R_{ij}(\tau) = \frac{1}{T_b}\int_0^{T_b} c_i(t)c_j(t-\tau)dt$$

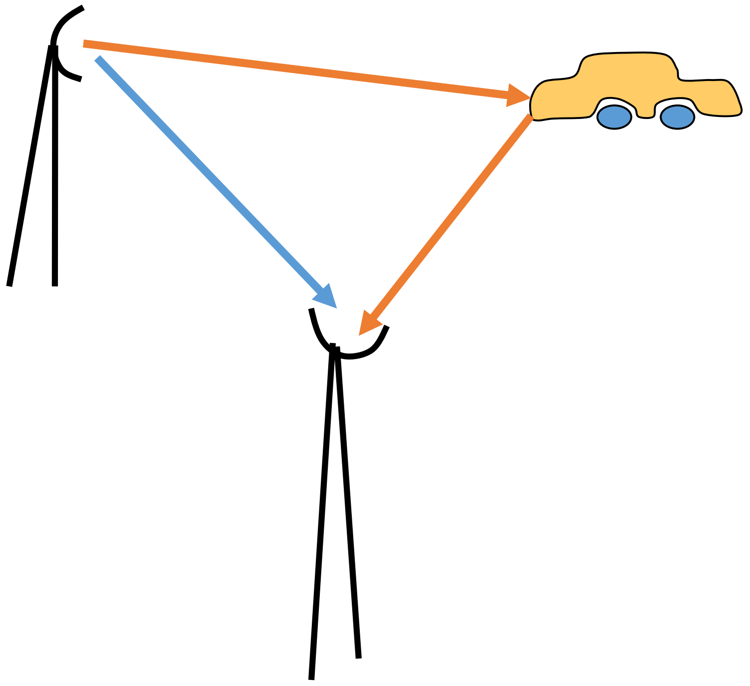
- **Auto-correlation**
  - The concept of determining how much similarity one set of data has with another
  - Range between –1 and 1
    - 1 The second sequence matches the first sequence
    - 0 There is no relation at all between the two sequences
    - -1 The two sequences are mirror images
- **Cross correlation**
  - The comparison between two sequences from different sources rather than a shifted copy of a sequence with itself

Inter-symbol Interference (ISI) Rejection

- Let us consider one message source m_1. The Transmitted signal s(t) over one bit interval T_b is:

$$s(t) = m_1 c_1(t)$$

- In wireless communications, the received signal consists of a direct component plus many reflected signals that arrive at a later time. Here we consider the direct signal plus one reflection: $r(t) = s(t) + s(t-\tau)$



- Received signal after de-spreading:

$$r(t)c_1(t) = m_1 c_1(t)c_1(t) + m_1 c_1(t-\tau)c_1(t)$$

- To demodulate m_1, we multiply r(t) by c_1 and integrate over T_b .

$$\int_0^{T_b} r(t)c_1(t)dt = \int_0^{T_b} c_1(t)[m_1 c_1(t) + m_1 c_1(t-\tau)]dt$$

$$V_0(T_b) = m_1 T_b + m_1 \int_0^{T_b} c_1(t)c_1(t-\tau)dt$$

Inter-symbol Interference (ISI) Rejection

$$V_0(T_b) = m_1 T_b + m_1 \int_0^{T_b} c_1(t) c_1(t-\tau) dt$$

$$V_0(T_b) = m_1 T_b + m_1 T_b R_{11}(\tau)$$

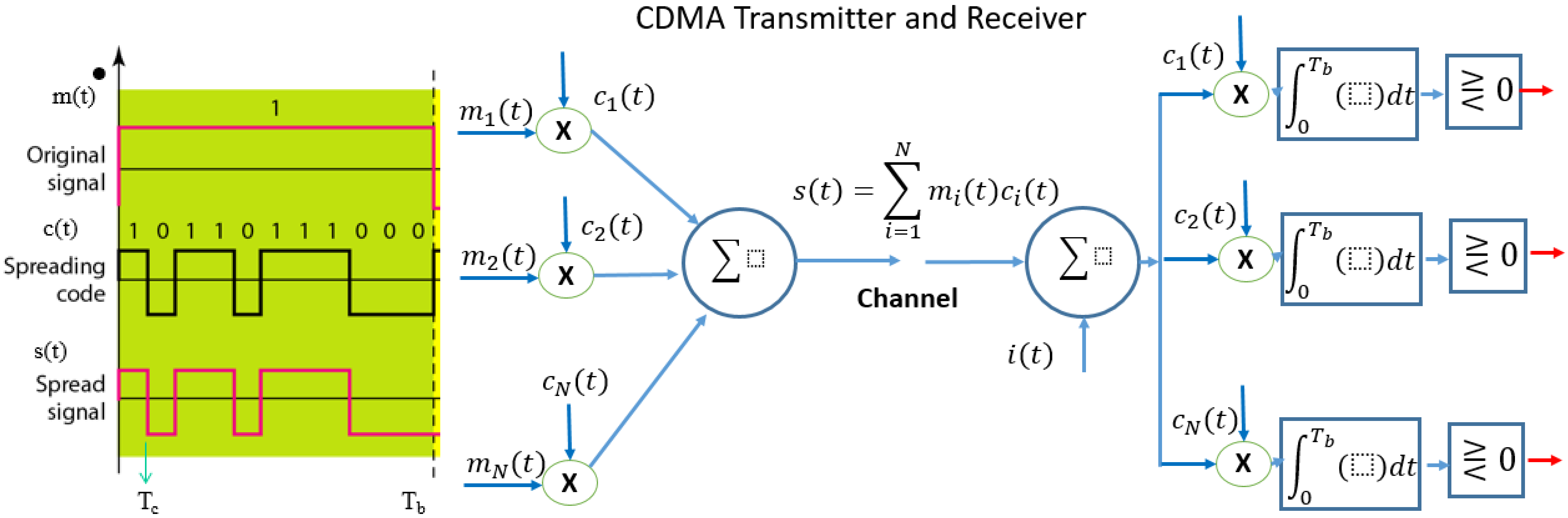
- The first term is the desired signal component, while the second is an inter-symbol interference (ISI) term arising from the reflected term introduced by the wireless channel.
- Ideally, to get rid of this term we require that

$$R_{ii}(\tau) = \begin{cases} 1 & \tau = 0 \\ 0 & \text{otherwise} \end{cases}$$

- **Remark 1**: This is one desired property of signature waveforms. When this condition holds, then ISI term = 0.
- **Remark 2**: Walsh-Hadamard sequences have poor autocorrelation functions. That is they do not satisfy the condition above. Therefore, one should search for better alternative signature waveforms.

Multi-access interference in CDMA

- In this lecture and the previous one, we address a number of issues related to CDMA:
  - **First**: How spread spectrum provides immunity from Jamming noise (last lecture)
  - **Second:** Inter-symbol interference in wireless transmission (last lecture)
  - **Third**: Multi-access interference (this lecture)
  - **Fourth**: Desired properties of autocorrelation and cross correlation functions of the spreading sequences (this lecture)



$T_b = N\ T_c$

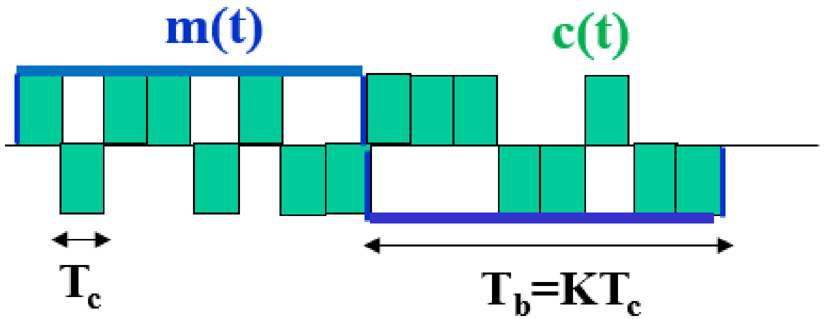
Signal to Interference Noise Ratio Improvement using the Spreading Sequence

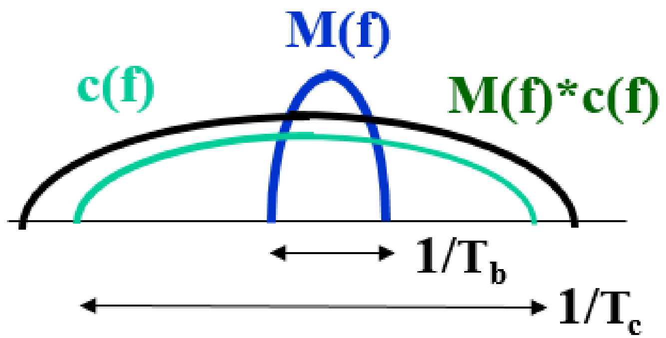
Signal to Noise Ratio without Spreading

Let P_M be the power in the message m(t) and

Let P_I be the power in the in the interfering signal i(t). So,

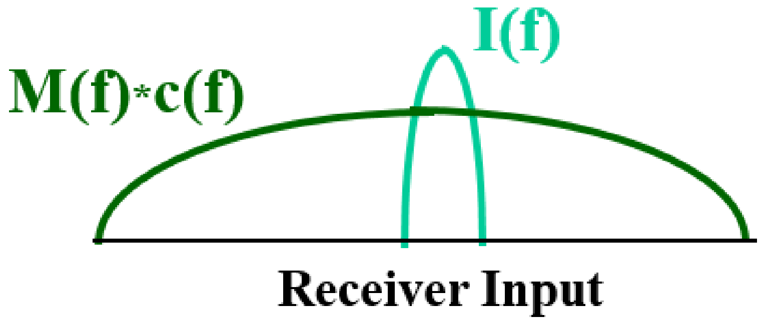
$$SNR_{without} = P_M/P_I$$

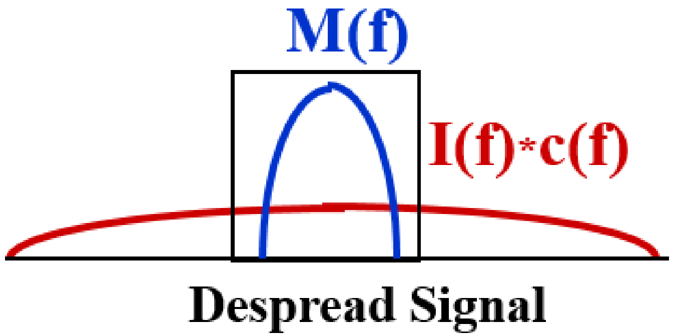




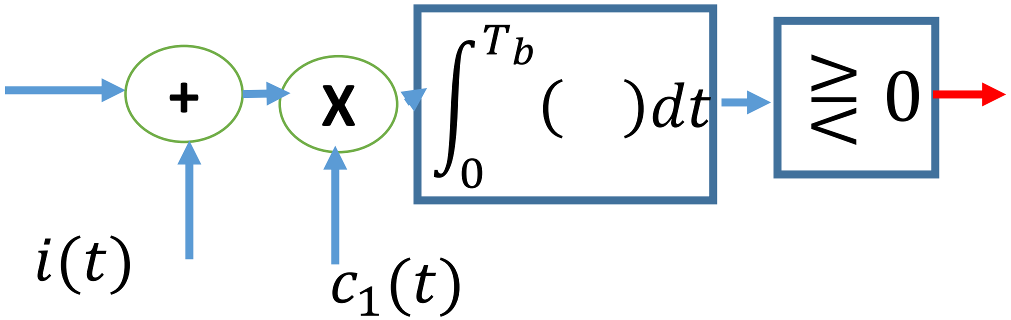
Signal to Noise Ratio without Spreading

$$SNR_{with} = \frac{P_M}{(P_I|N)} = N(P_M/P_I)$$





$$s(t) = \sum_{i=1}^{N} m_i(t)c_i(t)$$



$$V_0(T_b) = m_1 T_b + \int_0^{T_b} i(t)c_1(t)dt$$

Two Problems in Wireless CDMA Systems

- Let us start by defining the two correlation functions

Autocorrelation Function

$$R_{ii}(\tau) = \frac{1}{T_b}\int_0^{T_b} c_i(t)c_i(t-\tau)dt$$

Cross Correlation Function

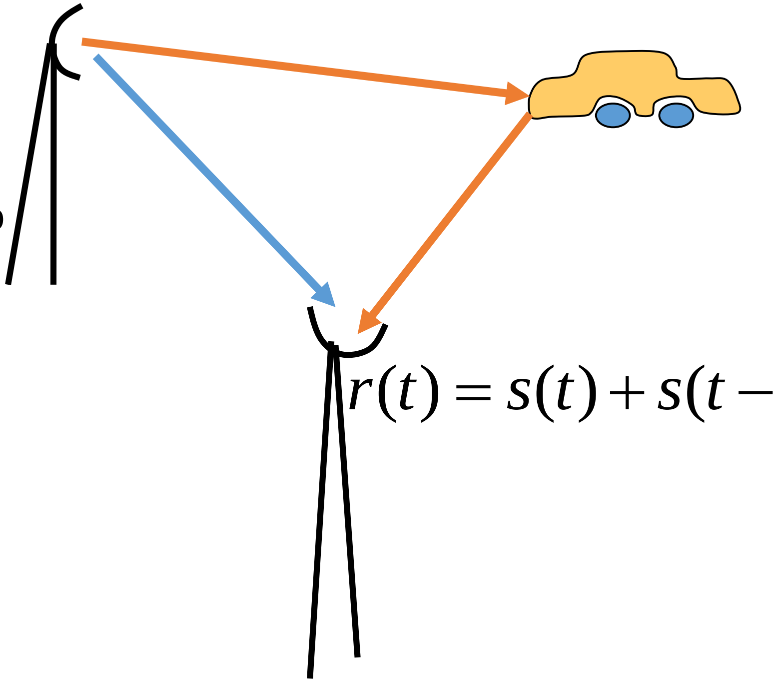
$$R_{ij}(\tau) = \frac{1}{T_b}\int_0^{T_b} c_i(t)c_j(t-\tau)dt$$

- **Auto-correlation**
  - Determines how much similarity a sequence of data has with a shifted version of itself.
  - Range between –1 and 1
    - 1 The second sequence matches the first sequence
    - 0 There is no relation at all between the two sequences
    - -1 The two sequences are mirror images
- **Cross correlation**
  - The comparison between two sequences from different sources rather than a shifted copy of a sequence with itself

Inter-symbol Interference (ISI) Rejection

- Let us consider one message source m_1. The Transmitted signal s(t) over one bit interval T_b is:

$$s(t) = m_1 c_1(t)$$



- To demodulate m_1, we multiply r(t) by c_1 and integrate over T_b

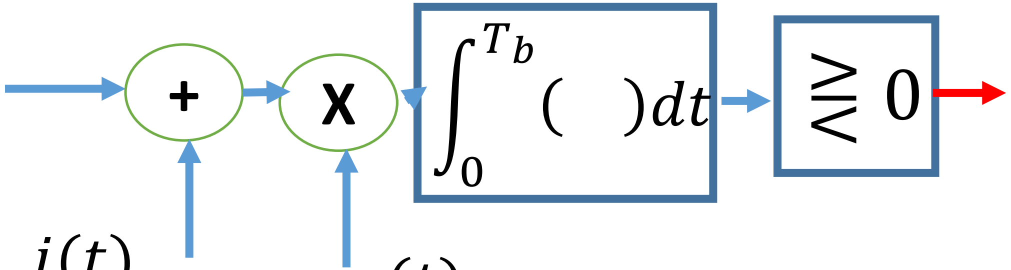
$$V_0(T_b) = m_1 T_b + m_1 \int_0^{T_b} c_1(t) c_1(t-\tau) dt$$

$$V_0(T_b) = m_1 T_b + m_1 T_b R_{11}(\tau)$$

- Ideally, to get rid of this ISI term we require that

$$R_{11}(\tau) = \begin{cases} 1 & \tau = 0 \\ 0 & \text{otherwise} \end{cases}$$

$r(t) = s(t) + s(t-\tau)$



$$V_0(T_b) = m_1 T_b + \int_0^{T_b} i(t) c_1(t) dt$$

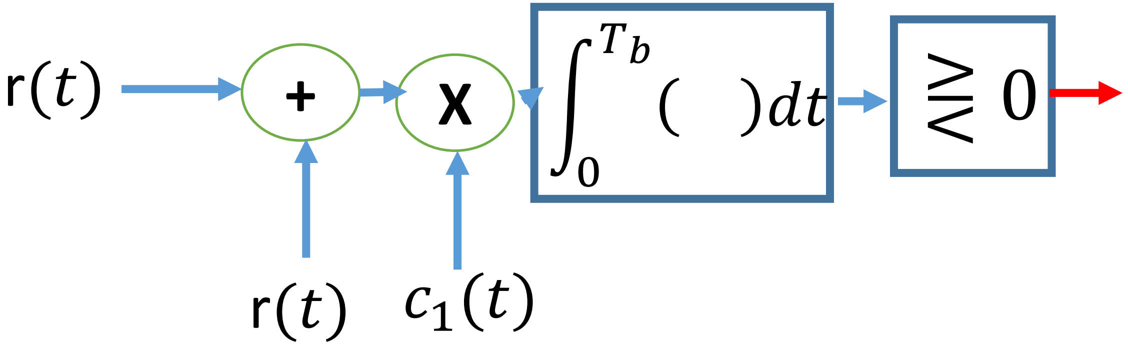
Multiple Access Interference (MAI) Rejection

- Received signal (at base station) from all users (for simplicity, no multipath or reflections are assumed) taking the path from user 1 to the base station as a reference

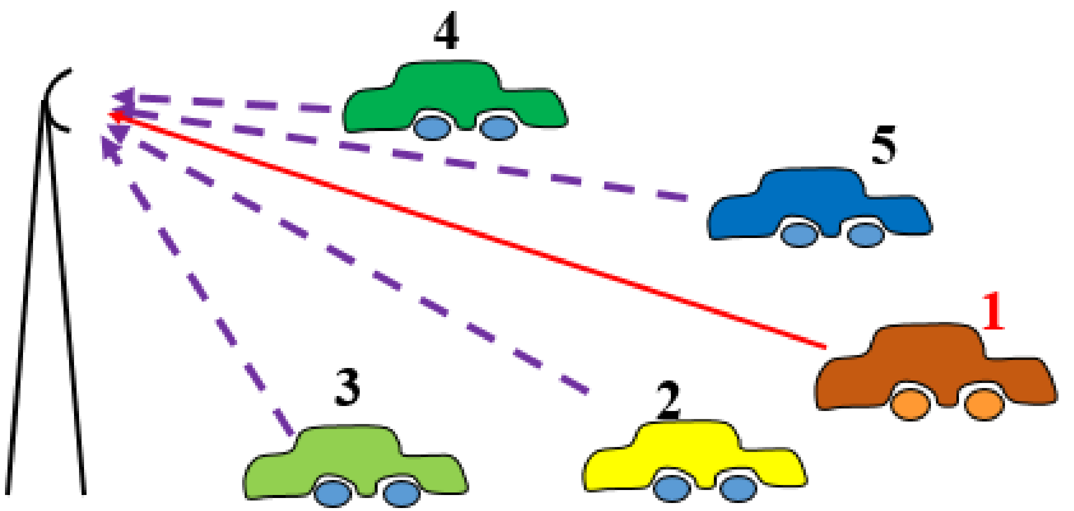
$$r(t) = m_1 c_1(t) + \sum_{j=2}^{M} m_j c_j (t - \tau_j)$$

- Received signal after de-spreading
- In the demodulator, this signal is integrated over a symbol time, so the output becomes

$$r(t)c_1(t) = m_1 (c_1(t))^2 + \sum_{j=2}^{M} m_j c_j (t - \tau_j) c_1(t)$$



- Assume N users (in the uplink from users to the base station) operating in the same frequency band, each using his own code $c_i(t)$.
- The signal from each user to the receiver (base station) suffers from a time delay which differs from user to user.
- Assume the base station wants to demodulate the signal of user 1
- **The other users will interfere with user 1. How?**



Multiple Access Interference (MAI) Rejection

- User 1 Received signal from all users (no multipath)

$$V_0(T_b) = m_1 T_b + m_j \sum_{j=2}^{M} \int_0^{T_b} c_j(t-\tau_j) c_1(t) dt$$

$$V_0(T_b) = m_1 T_b + m_j T_b \sum_{j=2}^{M} R_{1j}(\tau_j)$$

- The first term is the desired signal component.
- The second term is the multiple access interference (MAI).
- Ideally, we should have, **$R_{ij}(\tau)=0$ for all τ**, that is, **MAI = 0** as we have seen using Walsh-Hadamard synchronous demodulation.
- Unfortunately, Walsh-Hadamard sequences give rise to large values of MAI. Therefore, if the received signal consists of many terms that have different delays, these sequences cannot be used.

Autocorrelation and Cross-correlation Requirements

Good signature waveforms (spreading codes) should have

a. An ideal autocorrelation function of the form:

$$R_{ii}(\tau) = \begin{cases} 1 & \tau = 0 \\ 0 & \text{otherwise} \end{cases}$$

This property removes inter-symbol interference (ISI) as we have seen before.

b. An ideal cross correlation function that is zero for all shifts, i.e.,

$$R_{ij}(\tau) = \begin{cases} 0 & \tau = 0 \\ 0 & \tau \neq 0 \end{cases}; \quad i \neq j \text{ for all } \tau$$

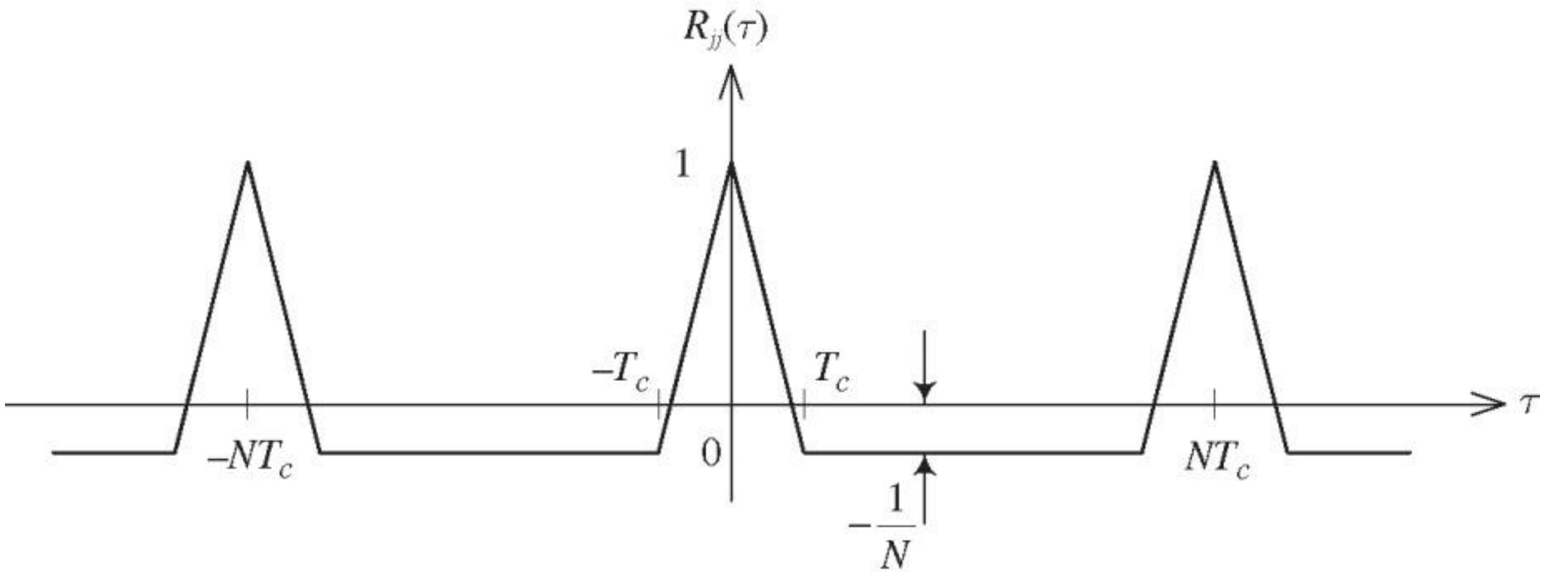
This property removes interference between users (MAI).

- Walsh-Hadamard sequences do not possess these properties.
- These ideal conditions are achieved if the sequences are chosen to be **random sequences**. But the problem with random sequences is that they have to be known to both the transmitter and the receiver, which is impossible. => cannot demodulate the signal!!
- What is the solution?

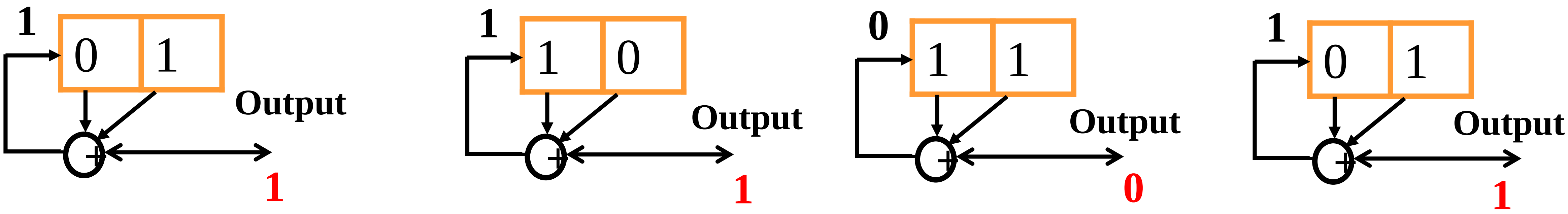
Pseudonoise (PN) Sequences

- The solution comes through the use of Pseudonoise (PN) sequences.
- These codes are periodic deterministic sequences that appear to be random.
- They are generated by a shift register (m flip flops, i.e., two state memory elements) and XOR
- **Maximum-length (ML) sequences**: *m*-stage shift register, periodic with period: $N = 2^m - 1$ bits
- Ideal PN sequences should be
  - Autocorrelation similar to white noise (high at τ=0 and low for τ not equal 0)
  - Orthogonal (Hence, they eliminate interference)
  - Random (provide security)

$$R(\tau) = \begin{cases} 1-(1+\frac{1}{N})\frac{|\tau|}{T_c} & |\tau| \le T_c. \\ -\frac{1}{N} & \text{otherwise} \end{cases}$$



Generating PN Sequences



- Take m=2
- Initial states (0,1)
- C_n = [1,1,0,1,1,0, . . .]
- Periodic with period N=3 bits.
- The all zero state is not permissible

At each pulse of the clock, the state of each flip flop is shifted to the next one. The PN sequence generated is determined by the number of shift registers, the initial state, and the Boolean function,.

Generating PN Sequences

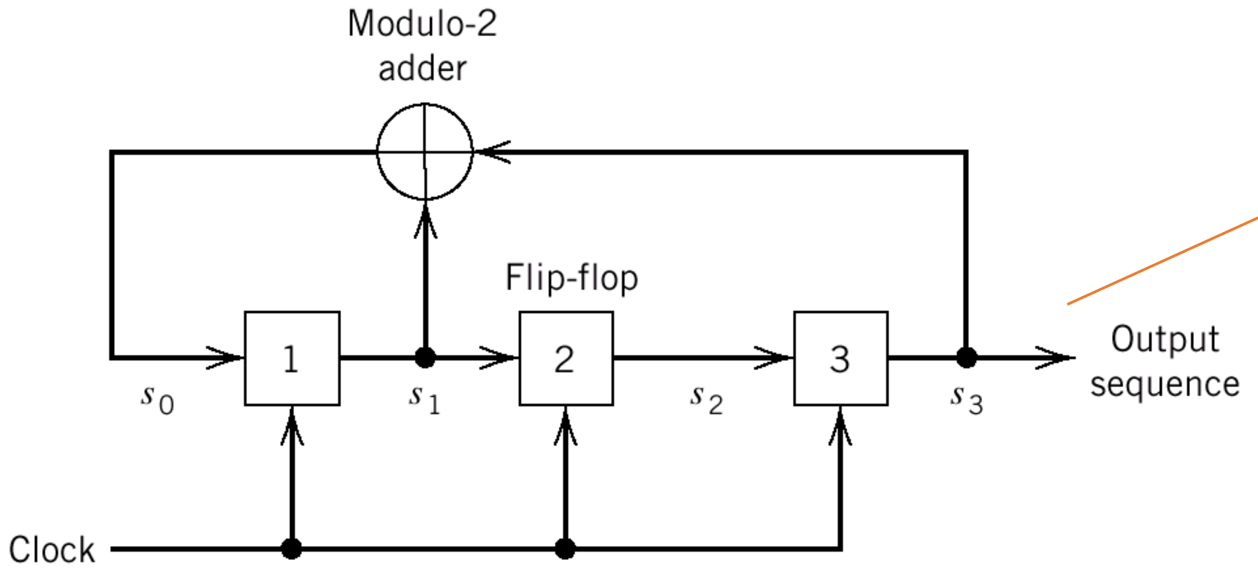
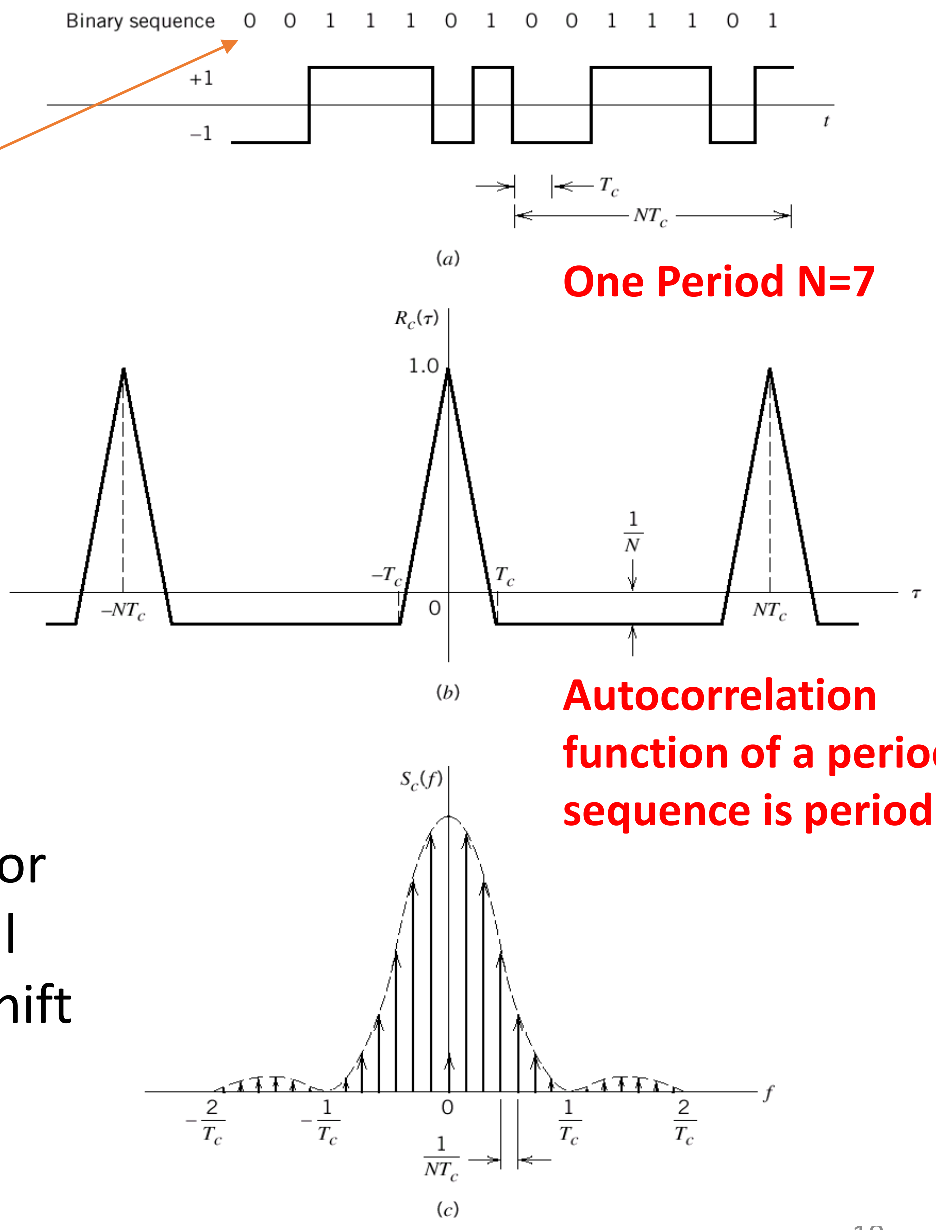


Figure 7.2 (Haykin):
Maximal-length sequence generator for $m = 3$.



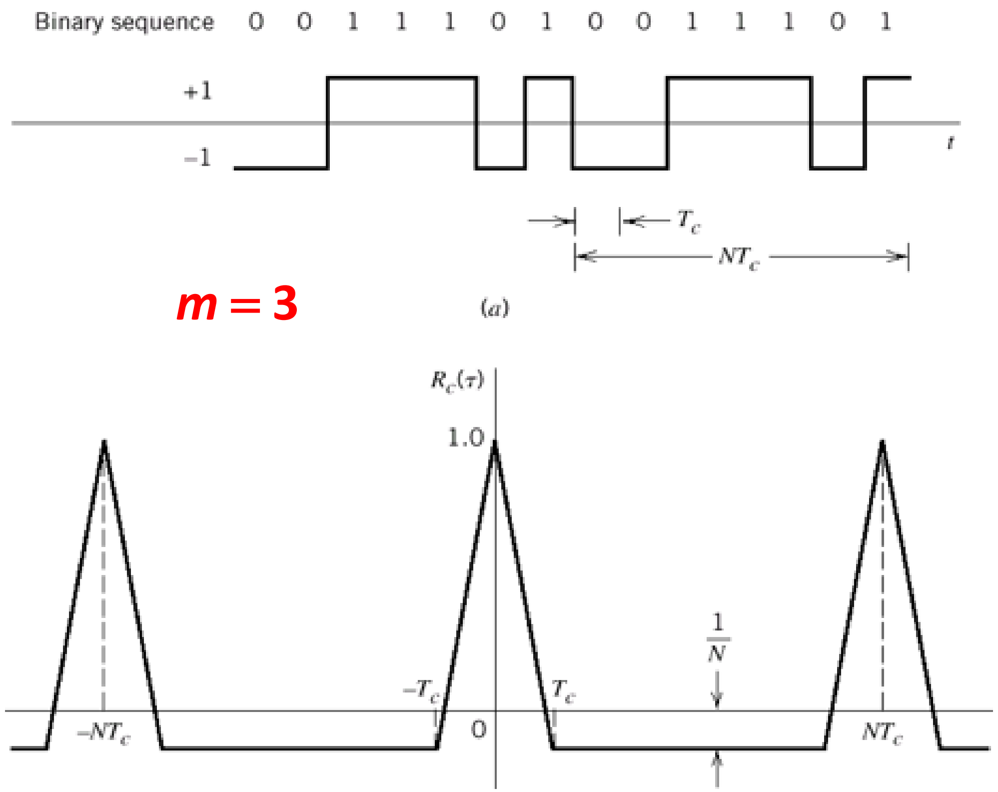
One Period N=7

Autocorrelation function of a period sequence is periodic

Figure 7.3 (Haykin)
(*a*) Waveform of maximal-length sequence for length $m = 3$ or period $N = 7$. (*b*) Autocorrelation function. (*c*) Power spectral density. All three parts refer to the output of the feedback shift register of Figure 7.2.

Properties of Maximal Length Sequences

- **Maximal length feedback shift register sequences have good properties**
  - **Has 2^{m-1} ones and 2^{m-1}-1 zeros. Therefore, in a long sequence, # of 1s and # of 0s are approximately equal. Therefore, the probability of a 0 is almost the same as the probability of 1. It looks like a random sequence**
  - **No DC component**
  - **Large period $(2^m-1)T_c$**
  - **The autocorrelation is small except when τ is approximately zero**
    - **ISI rejection.**



$m = 3$

Problem 1: Cross-correlations with other *m*-sequences generated by different input sequences can be quite high (Poor cross correlation)
Problem 2: jammer can determine the feedback connections by observing only 2*m-1 chips from the* PN sequence.
In practice, Gold codes or Kasami sequences which combine the output of m-sequences are used.

Gold Sequences

- Gold sequences constructed by the XOR of two m-sequences with the same clocking
- Only simple circuitry needed to generate large number of unique codes
- In this example, two shift registers generate the two m-sequences and these are then bitwise XORed.
- Codes have well-defined cross correlation properties
- These will be a subject of another lecture.